Coherent Pound–Drever–Hall Technique for High Resolution Fiber-Optic Sensors at Low Probe Power

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Abstract—The Pound–Drever–Hall (PDH) technique has been widely adopted in high-resolution fiber-optic sensors, but its performance degrades as the probe power drops. In this work, we develop a coherent PDH technique for detection of very weak probe light, in which the probe beam is coherent detected with a strong local oscillator. Assisted with an analog frequency doubler and a band-pass filter, the configuration of proposed coherent PDH technique is highly compatible with classical PDH technique. The influence of fiber dispersion is also assessed. In the demonstrational experiments, the signal-to-noise ratio of the extracted PDH signal is dramatically improved compared with classical PDH technique, especially under weak probe power. Using a π-phase shifted fiber Bragg grating as the sensing element, a π-order strain resolution is achieved at a low probe power down to –43 dBm, which is about 15-dB lower compared with classical PDH technique. The proposed technique has great potentials in high-resolution large-scale fiber sensor networks.

Index Terms—Fiber gratings, optical sensors, optical signal detection, Pound–Drever–Hall technique.

I. INTRODUCTION

THANKS to the advantages of small size, low cost, high resolution, immunity to electromagnetic interference, and the capability of remote and multiplexed sensing, optical fiber grating based sensors have been widely adopted in measurement of parameters including strain, temperature, pressure, displacement, accumulation, and vibration [1]. The measurable can be linearly encoded in the grating’s spectrum, which is an absolute character immune to the power losses of the fiber components and links, so high performance is expectable even under harsh environment. Compared the uniform fiber Bragg grating (FBG), the fiber Fabry–Perot resonator formed by a pair of high reflection FBG written on a piece of fiber [2] and the π-phase shifted FBG (π-PSFBG) [3] have much narrower resonance spectra, and are more proper in applications where high resolution is required. The resolution is one of the most important parameters for the grating sensors, which is mainly determined by the precision in the detection of the resonant wavelength of the gratings.

Many types of high resolution interrogation technologies for fiber grating sensors have been developed. Using a narrow linewidth laser to interrogate the FBG at the edge of the reflection peak has reached a strain resolution down to 45 με/√Hz at 3 kHz [4]. The transmission spectrum of the FBG is also employed for strain sensing. Using special designed FBG with enhanced slow-light transmission peaks, strain resolution of 280 fs/√Hz at 23 kHz was reported [5]. The above direct power detection method is simple with high resolution, but the measurement range is limited by the linear range of the spectrum. To extend the measurement range, techniques of tracking the resonant wavelength of the fiber gratings using a tunable narrow linewidth laser is developed. The key work to improve the resolution is the precision in the measurement of the wavelength difference between the resonator and the laser, for which the Pound-Drever-Hall (PDH) technique has been widely adopted.

The PDH is initially developed for laser frequency stabilization [6], and now it plays an important role not only in laser frequency stabilization [7]–[10], but also in gravitational-wave detection [11], [12], optical comb generation [13], and parameter measurement of Fabry-Perot interferometers [14], [15]. The PDH technique has also been widely adopted for the interrogation of fiber grating sensors [16], [17]. Broadband strain resolution of 2 ps/√Hz is realized by using fiber Fabry–Perot resonator interrogated by PDH technique [16]. Even higher resolution of 120 fs/√Hz at 2 Hz is reported by using PDH technique and an optical frequency comb source [17]. Sideband interrogation technology is also developed to enable quasi-static strain sensing with large dynamic range, with which the resonant frequency of the fiber gratings is decoupled from the laser frequency [18], [19].

Large scale sensor array containing tens to hundreds of high resolution sensing gratings are demanded in applications such as hydrophone array, microseisms monitoring for oil well, crustal deformation monitoring, etc. Both wavelength division multiplexing (WDM) scheme [20] and time-division multiplexing (TDM) scheme [21] has been developed in cooperation with the PDH technique. As the scale of sensor array increases, the insertion loss of the fiber cable and components leads to a very weak power at the receiver, especially in the TDM based sensor array because of the parallel structure of the gratings [21]. Although the extracted signal (PDH signal) demodulated from PDH technique is immune to the low frequency intensity noise of the fiber link because the signal under detection is modulated to radio frequency (RF) region, the amplitude of the PDH signal is proportional to the power of the probe light. As the probe
light power drops due to large loss, the signal to noise ratio (SNR) of the PDH signal degenerates, and hence the resolution of measurement gets worse. The in-line optical fiber amplifier can partly compensate the power loss of the fiber link, but its applicability is limited due to the complexity such as extra electrical power supply system.

As an effective method for the detection of weak light, coherent detection is very attractive for the interrogation of large scale fiber grating sensor array with high resolution, if it is compatible with the PDH technique. In this paper, we developed a coherent PDH technique, in which for the first time the coherent detection is combined with the PDH technique. Heterodyne detection is employed with a strong local oscillator (LO) to generate the beat signal at low probe power. The frequency components involved in PDH technique are all reserved after coherent detection, except that their frequencies shift from optical domain to RF region. To extract the PDH signal, an analog frequency doubler and a band-pass filter are employed, and the configuration is highly compatible with the classical PDH configuration. By adjusting the power of the LO, error signal with high SNR is obtained even at low probe light power. Theoretical analysis also reveals that the dispersion of the fiber link is ignorable for the common applications. In the demonstrational experiments, a $\epsilon$-order strain resolution is achieved with the probe light power down to $\pm 43$ dBm, which is about 15 dB lower compared with classical PDH technique. Due to the remarkably extension of probe light power range, the number of sensing gratings can be at least increased fourfold. The proposed technique has great potentials in high resolution large scale fiber sensor networks.

II. PDH TECHNIQUE WITH COHERENT DETECTION

A. Classical PDH Configuration

The schematic of fiber-optic sensor using PDH technique is illustrated in Fig. 1. A $\pi$-phase shifted FBG ($\pi$-PSFBG) is used as the sensing element, which has a phase shift of $\pi$ at the center of the grating. With this phase shift, the $\pi$-PSFBG can be conceptually regarded as a Fabry-Perot cavity formed by two FBG mirrors, and it exhibits a narrow notch ($< 1$ pm) within its high reflection band [22], much narrower than the bandwidth of an uniform FBG (>< 50 pm). Such a sharp spectrum helps to achieve high precise detection of resonance frequency and hence the high strain resolution. Here the sensing element can also be an actual fiber Fabry-Perot resonator formed by a pair of high reflection FBGs, which has an even narrower resonant spectrum but a longer length (tens to hundreds of centimeters).

In the PDH technique, the probe beam from a narrow linewidth laser source is phase modulated by a phase modulator (PM) before the interrogation of the $\pi$-PSFBG. After the PM the form of the probe beam is:

$$E_{in} = E_0 e^{i\omega t} + \epsilon e^{i\beta \sin \Omega_M t},$$

(1)

where $E_0$ is the amplitude of the incident beam, $\omega$ is its center frequency, $\Omega_M$ and $\beta$ are the phase modulation frequency and modulation depth, $J_0(\beta)$ and $J_1(\beta)$ are the zero-order and the first-order Bessel coefficients, respectively. The frequency components of the probe light are shown as the purple lines in Fig. 2(a). The probe beam is then leaded to the $\pi$-PSFBG via a fiber circulator (CIR). A variable optical attenuator (VOA) is used to control the intensity of the probe light, simulating the loss of the fiber link and optical components.

In classical PDH technique, the reflected probe light by the $\pi$-PSFBG is directly received by a photo-detector (PD), as shown by the inset in Fig. 1. The instantaneous frequency of the probe light is dithered by the phase modulation, which results in a power modulation of the reflected lightwave, because the reflection of the $\pi$-PSFBG is highly depended on the frequency of the probe. A lock-in amplifier (LIA) is employed to detect the amplitude and phase of the power modulation, and the PDH signal is extracted [11].

For the case that the phase modulation frequency $\Omega_M$ is smaller than the bandwidth of the $\pi$-PSFBG $\delta\nu$ ($\Omega_M < \delta\nu$, so-called slow modulation in PDH technique), the PDH signal near resonance is expressed as [11]:

$$\epsilon_d(\omega) = -2R_{PD} P_0 J_0(\beta) J_1(\beta) \frac{d|F(\omega)|^2}{d\omega} \Omega_M,$$

(2)
The frequency of LO is shown as the red line in Fig. 2(a). The different time delay between the LO and reflected probe beam. The reflected probe by the -PSFBG is coherent detected with the LO on a balanced photo-detector (BPD), and Fig. 2(b) illustrates the beat frequency components. A band-pass filter (BPF) is employed to extract the frequency components around \( \Omega_0 \) (in the form of complex after Hilbert’s transform, without considering the reflection coefficient of the \( \pi \)-PSFBG):

\[
S \approx R_{BPD} E_l e^{i(\omega t - \Omega_0 t + i\phi(t))} \left[ J_0 (\beta) + J_1 (\beta) e^{i\Omega_M t} - J_1 (\beta) e^{-i\Omega_M t} \right],
\]

where \( R_{BPD} \) is the responsivity coefficient of the balanced photo-detector. The frequency components are shown in Fig. 2(b). Equation (4) has the same form as (1) except for the coefficients and center frequency. Such a signal reserves the necessary information for PDH signal extraction, and can be demodulated with proper configuration.

### C. Demodulation for Coherent PDH Technique

The beat signal from the balanced photo-detector can be demodulated either digitally or assisted with analog circuits. For the fully digital demodulation method, the beat signal can be sampled under Nyquist–Shannon sampling theorem with a high speed data acquisition device, and the PDH signal is directly calculated out using the components in (4) [11]. In this work, we develop an analog devices assisted demodulation configuration, with which the expensive high speed data acquisition device is avoided, and the configuration is highly compatible with the classical PDH configuration.

In the classical PDH technique, the photo-detector works as a mixer with which the different optical frequency components combine to generate the beat signal at \( \Omega_M \), and the PDH signal is extracted from the beat signal using phase detection. In the proposed configuration, a frequency doubler is employed to carry out the interference among the beat frequency components, just as the role of the photo-detector in the classical PDH technique. The frequency components after the frequency doubler is shown in Fig. 2(c), where the component at \( \Omega_M \) is in the same form as the signal from the photo-detector in classical PDH technique. Finally, a lock-in amplifier is employed to detect the amplitude and phase of the \( \Omega_M \) component, which is the PDH signal:

\[
\epsilon_{co} (\omega) = 2k_e R_{BPD}^2 P_0 P_l J_0 (\beta) J_1 (\beta) \frac{d|F(\omega)|^2}{d\omega} \Omega_0, \quad (5)
\]

where \( P_0 \equiv |E_0|^2 \) is the power of LO, \( k_e \) is the coefficient of frequency doubler including the insertion loss of the band-pass filter. It is found that the phase deviation term \( \phi(t) \) in (3) does not appear in the extracted PDH signal, because this term has the same influence on all the frequency components in (4), and it vanishes in the process of frequency mixing. In other words, the proposed configuration is immune to the phase noise of the LO.

From the comparison between (5) and (2), it is found that compared with the classical PDH technique, the magnitude of PDH signal in coherent PDH technique is magnified by a factor of:

\[
M = \frac{k_e R_{BPD}^2 P_l}{R_{PD}}. \quad (6)
\]

By increasing the power of LO, the amplitude of the PDH signal can be remarkably amplified, especially in case of weak probe beam.

### D. The Influence of Dispersion

The PDH signal originates from the interferences among the carrier and sidebands (mainly the 1st order sidebands) of the probe. The influence of dispersion of fiber link should be assessed if the PDH technique is used in a large scale sensor network over long distance. The PDH technique is immune to the linear dispersion of the fiber, because the additional phase shift caused by linear dispersion is proportional to laser frequency,
and the phases of the components in (4) remain symmetrical to the carrier. The high orders dispersion, however, will destroy the symmetry of the phases, and results in distortion of the PDH signal. Considering the dispersion, (4) should be rewritten as:

\[
S \propto \left[ J_0(\beta) e^{i\Phi_0} + J_1(\beta) e^{i(\Omega M t + \Phi_{-M})} \right] - J_1(\beta) e^{-i(\Omega M t + \Phi_{-M})},
\]

where \( \Phi_0, \Phi_M \) and \( \Phi_{-M} \) are the phase shift of the carrier and sidebands traveling through the fiber link respectively. The asymmetry of phases is:

\[
\Delta \Phi = \Phi_M + \Phi_{-M} - 2\Phi_0 = L\beta_2 \Omega_M^2
\]

where \( \beta_2 \) is the group-velocity dispersion (GVD) of the fiber, \( L \) is the length of fiber. For the typical single mode fiber, the dispersion parameter is around \( D = 20 \text{ ps/nm/m} \), and the GVD is calculated to be \( \beta_2 = 25500 \text{ fs}^2/\text{m} \). When the fiber length is 50 km and the modulation frequency is 2 GHz, the phase asymmetry is calculated to be 0.20 rad. Fig. 3 illustrates the distorted PDH signal with the phase asymmetry of 0.2. The shape of the curve is almost unchanged, but the vertical offset of the curve deviates from zero.

The offset in Fig. 3 will result in a constant drift to the measured resonance frequency of the \( \pi \)-PSFBG. As shown by (8), the phase asymmetry depends on fiber length and frequency of phase modulation. For a given tolerable level of phase asymmetry, the allowed modulation frequency \( \Omega_M \) is restricted by the fiber link length in theory. As shown in Fig. 4, the phase asymmetry is ignorable small (0.001) with phase modulation of -20 dBm. The reflected lightwave by the \( \pi \)-PSFBG has a narrow linewidth laser (NKT, E15) is used as the light source, and its frequency is tuned to sweep around the resonance frequency of the \( \pi \)-PSFBG. The \( \pi \)-PSFBG has a narrow resonant frequency bandwidth with full-width at half-maximum (FWHM) of 70 MHz. The probe beam is phase modulated by a sinusoidal wave with frequency of \( \Omega_M = 7 \text{ MHz} \), and the modulation depth \( \beta \) is around 1.5. The LO passes through an acousto-optic modulator (AOM) with driving frequency of 80 MHz. A polarization controller (PC) is used to adjust the polarization state of the LO. The reflected light from the \( \pi \)-PSFBG is coherent detected with the LO light using a balanced photodetector (Thorlabs, PD460C). Then a band-pass filter is used to extract the beat signal around 80 MHz. An analog multiplier (Analog Inc., AD835) is used as the frequency doubler (FD). Since the input voltage range of the multiplier is \( \pm 1 \text{ V} \), a variable attenuator is used to adjust the amplitude of the beat signal in experiments. The output of the multiplier is connected to a lock-in amplifier (LIA) to extract the PDH signal. The demodulated PDH signal is received by an analog-to-digital converter (ADC).

### III. Experimental Results and Discussions

#### A. Experimental Setup

The proposed coherent PDH technique is verified in experiments. The block diagram of the system setup is the same as Fig. 1. A narrow linewidth laser (NKT, E15) is used as the frequency doubler of \( \pi \)-PSFBG. As shown by (8), the magnification factor is approximately 1.5. The LO passes through an additional variable attenuator of 10 dB, and the coefficient of the frequency doubler of \( k_e = 1 \text{ V}^{-1} \), the magnification factor is calculated to be \( M = 14.4 \), which agrees well with the experimental results.

#### B. Improvements of Magnitude and Signal-to-Noise Ratio

Although the PDH technique is commonly implemented with close feedback loop in applications like laser frequency stabilization, this demonstration experiment it works in open loop without quick feedback control for simplification. The laser frequency is tuned to sweep around the resonant frequency of the \( \pi \)-PSFBG, and the PDH signal is extracted for analysis. At first, the classical PDH technique is tested at a weak probe power of -20 dBm. The reflected lightwave by the \( \pi \)-PSFBG is directly detected by a photo-detector (Thorlabs, PDA10CS). The same lock-in amplifier is used to extract the PDH signal, which is shown in Fig. 5(a). The signal extracted from the coherent PDH technique with the same probe power and an LO of 0.8 dBm is shown in Fig. 5(b). The amplitude of the coherent PDH signal is amplified by 15.4 times according to the values in the figure. With the parameters of the devices: \( R_{BPD} = 30 \text{ kV/W}, R_{PD} = 0.75 \text{ kV/W} \), the total insertion loss of the BPF and the additional variable attenuator of 10 dB, and the coefficient of the frequency doubler of \( k_e = 1 \text{ V}^{-1} \), the magnification factor is calculated to be \( M = 14.4 \), which agrees well with the experimental results.
Fig. 5. The Extracted PDH signals from (a) classical PDH technique and from (b) coherent PDH technique. The probe power is $-20$ dBm for both case. The power of local oscillator in coherent PDH technique is 0.8 dBm.

The coherent PDH technique can remarkably magnify the amplitude of the PDH signal, while the noise level of the electronics devices such as photo-detector remains the unchanged. So the improvement of signal-to-noise ratio (SNR) of the demodulated PDH signal is expected, and it is verified in experiments. Here the SNR is defined as the ratio of the peak-peak value of the PDH signal around resonance frequency of the $\pi$-PSFBG $V_{p-p}$ in Fig. 5, and the standard deviation of the curve far away from the resonance (\(\sigma\) in Fig. 5). In this test, the power of LO increases from $-14$ dBm to $-6$ dBm, while the probe power remains $-20$ dBm. Due to the limited working range of the analog multiplier, the insertion loss of the attenuator is optimized at different LO powers. At each LO power, the measurement of SNR is repeated for 50 times, and the averaged SNR is shown in Fig. 6. As the power of LO increases, the SNR also increases from 20.8 dB to 36.7 dB, showing that the SNR benefits from the large magnification factor $M$. Obviously the SNR can’t be infinitely improved by increasing the LO power, because the inherent noise like shot noise of the probe light and LO is simultaneously magnified. This is the reason that the curve in Fig. 6 tends to flat at stronger LO power.

C. Improvement of Strain Resolution

The performance of coherent PDH technique is then experimentally demonstrated in a strain sensor. The sensing $\pi$-PSFBG is in relaxation without strain applied to assess the resolution. The laser frequency is tuned sweeping around the resonance frequency of the $\pi$-PSFBG, and each sweep will produce a PDH signal as shown in Fig. 5(b). The wavelength at cross-zero point of the curve indicates the resonance frequency (wavelength) of the $\pi$-PSFBG. It should be a constant in ideal situation, but actually it fluctuates due to noises, and the standard deviation is regarded as the resolution of the sensor. At each probe power level, the measurement is repeated for 50 times in 1 second, and then the strain resolution is calculated from the standard deviation of the wavelength with the strain sensitivity of 112.83 kHz/n\(\varepsilon\).

Fig. 7(a) is an example of measured results when the power of the probe light is $-25$ dBm. According to the standard deviation of the measured strain, the strain resolution of classical PDH technique is 6.3 nc. With a LO light power of 0.8 dBm, the strain resolution of coherent PDH technique is improved to 1.3 nc thanks to the much better SNR of the PDH signal.

The strain resolution with classical and coherent PDH techniques are compared at different probe power from $-10$ dBm to $-43$ dBm, with a step of 3 dBm. The comparison is shown in Fig. 7(b). When the probe power is larger than $-13$ dBm, the strain resolutions of both techniques are almost the same, showing that the SNR of PDH signal is not the bottleneck of strain resolution. The assessed strain resolution is better than 1 nc, mainly limited by the laser frequency noise and ambient interference.

As the probe power drops, the resolution of classical PDH technique is more sensitive to the probe power. When the probe power is lower than $-28$ dBm, the PDH signal of the classical PDH technique is completely merged in noise, while the coherent PDH technique based sensor can still work effectively with a strain resolution of 2.2 nc at $-31$ dBm. The minimum detectable probe power of coherent PDH technique is $-43$ dBm with a strain resolution of 10.7 nc, 15 dB lower than the classical PDH technique. The much lower detectable power indicates that sensor networks with coherent PDH technique could have much larger array scale and longer distance.

IV. CONCLUSION

In conclusion, we have proposed a coherent PDH technique that has much better performance at very weak probe power compared with classical PDH technique. With this technique, a high resolution fiber-optic strain sensor is developed, and a nc-order strain resolution is achieved experimentally at a probe power down to $-43$ dBm, which is 15 dB lower compared with
classical PDH technique. The proposed technique is also compatible with the multiplexing techniques such as time-division multiplexing. Due to the dramatically improved performance of weak light signal detection, the proposed coherent PDH technique has great potentials in long distance and large scale high resolution grating sensor networks.

REFERENCES


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